

Roll No. :

BCA-1005

BCA (Semester-I) (NEP) Examination, 2025-26

(Back Paper)

MATHEMATICS-I

[Paper : Fifth]

Time : 2 Hours]

[Maximum Marks : 75

Note : 1. This paper consists of three sections- A, B and C.
Attempt questions from **all** sections as directed.

2. The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

3. "B" copy will not be provided.

Section-A

Short Answer Type Questions

Note : **All** questions are **compulsory**. Each question carries
5 marks.

[9×5=45]

BCA-1005/830

(1)

Turn Over

1. (a) Find the value of c using Lagrange's mean value theorem for $f(x) = (x-1)(x-2)(x-3)$ in interval $[0, 4]$.

(b) If $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \\ 1 & 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 4 \\ -2 & 1 & 5 \end{bmatrix}$,

then find AB .

(c) Evaluate $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$.

(d) If $y = x^x$, then find $\frac{dy}{dx}$.

(e) Evaluate indefinite integral $\int \frac{1}{\sqrt{9+x^2}} dx$.

(f) If A, B are square matrices, such that $A^2 = A$ and $B^2 = B$, show that $(AB)^2 = AB$, if A, B commute.

(g) Find the value of a for which the vectors $\vec{A} = a\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = -\hat{i} + 5\hat{j} + a\hat{k}$ are perpendicular.

- (h) Find the value of the determinant :

$$\begin{vmatrix} 1 & 3 & -1 \\ -2 & 1 & 2 \\ 4 & 5 & 1 \end{vmatrix}$$

- (i) Prove that :

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}.$$

Section-B

Long Answer Type Questions

Note : Attempt **any one** question from the following. Each question carries 15 marks. [1×15=15]

2. (a) Find the first derivative of $y = \frac{e^{2x}}{e^{2x} + 1}$.

(b) Evaluate $\lim_{x \rightarrow 0} \frac{2x^2 \sin \frac{1}{x}}{e^x}$.

- (c) Prove that the function $f(x) = \frac{|x|}{x}$ for $x \neq 0$, $f(0) = 0$ is continuous at all point except $x = 0$.

3. (a) Show that the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ is a solution of the equation $A^2 - 5A + 7I_2 = 0$, where I_2 is 2×2 unit matrix.

(b) Find Maclaurin's series expansion of function $\log(1 + e^x)$ about the point $x = 0$.

(c) Investigate maxima and minima of

$$f(x) = 2x^3 - 21x^2 + 36x - 20.$$

4. (a) Write statement of Cayley Hamilton theorem and find characteristic equation of matrix

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 3 & 1 & -3 \end{bmatrix}.$$

(b) Verify Cayley-Hamilton theorem for the

$$\text{matrix: } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}.$$

(c) If $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and

$$(A + B)^2 = A^2 + B^2 + 2AB, \text{ find } a \text{ and } b.$$

5. (a) Find the derivative of $\frac{1}{(x+1)(x+2)(x+3)}$.

(b) Find A^{-1} , where $A = \begin{bmatrix} 4 & 7 & 2 \\ 3 & 6 & 1 \\ 2 & 5 & 1 \end{bmatrix}$.

(c) Evaluate $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$ using LHL and RHL method.

Section-C

Long Answer Type Questions

Note : Attempt **any one** question from the following. Each question carries 15 marks. [1×15=15]

6. (a) If $f(x) = x^3 + 8x^2 + 15x - 24$, calculate the value of $f\left(\frac{11}{10}\right)$ by the application of Taylor's series.

(b) Evaluate indefinite integral $\int \frac{\sin^4 x}{\cos^8 x} dx$.

7. (a) Find a vector of magnitude 15, which is perpendicular to both the vectors $\vec{a} = 4\hat{i} - \hat{j} + 8\hat{k}$ and $\vec{b} = -\hat{j} + \hat{k}$.

(b) Evaluate integral by parts $\int \frac{\log x}{x^2} dx$.

8. (a) Find $\frac{dy}{dx}$ when $x^y + y^x = a$.

(b) Evaluate integral $\int x^2 \tan^4 x^3 \sec^2 x^3 dx$.

9. (a) Define scalar and vector triple product. Find the value of λ so that the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{j} + \lambda\hat{k}$ are coplanar.

(b) Verify Rolle's theorem for the function $f(x) = (x-3)^4(x-4)^5$ in $[3, 4]$.

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